Part 1, MULTIPLE CHOICE, 5 Points Each

1 The probability distribution for a random variable X is given below. What is the variance of X (denoted σ^2 or $\mathrm{Var}(X)$)?

k	$\Pr(X=k)$	$k\,Pr(X=k)$	$k-\mu$	$(k-\mu)^2$	$(k-\mu)^2 Pr(X=k)$
−2	.3	−.6	−2	4	1.2
−1	.1	−.1	−1	1	.1
0	.2	0	0	0	0
1	.1	.1	1	1	.1
2	.3	.6	2	4	1.2
		$0=\mu$			$\sigma^2 = 2.6$

(a) 0 $\quad$ $\cancel{(b)}$ 2.6 $\quad$ (c) 10 $\quad$ (d) 2 $\quad$ (e) 1.5

$$\sigma^2 = \sum (k-\mu)^2 Pr(X=k)$$

2 Let Z be a random variable with a standard normal distribution. What is $Pr(Z \geq -1)$? (Use the attached tables)

(a) .1587 $\quad$ (b) .5 $\quad$ $\cancel{(c)}$ .8413 $\quad$ (d) .5398 $\quad$ (e) .4602

$$= 1 - Pr(Z \leq -1)$$

$$= 1 - .1586 = .8413$$

1

3 Which of the pairs of values for x and y given below is in the feasible set for the set of inequalities:

$$x + 2y \geq 7$$
$$2x - y \geq 4$$
$$y \geq 0, \quad x \geq 0.$$

(a) $x = 1, \; y = 3$ $\quad$ (b) $x = 0, \; y = 4$ $\quad$ (c) $x = 2, \; y = 1$

(d) $x = 3, \; y = -2$ $\quad$ (e) $x = 7, \; y = 1$

	$x=1, y=3$	$x=0, y=4$	$x=2, y=1$	$x=3, y=-2$	$x=7, y=1$
$x+2y \geq 7$	✓	✓	✗	✗	✓
$2x-y \geq 4$	✗	✗			✓
$x \geq 0$					✓
$y \geq 0$					✓

4 Which of the following give the co-ordinates of the point of intersection of the two lines:

Standard form

$$5x - y = 2 \quad \rightarrow -y = 2 - 5x \rightarrow y = 5x - 2$$
$$-8x + 4y = 4 \quad \rightarrow 4y = 4 + 8x \rightarrow y = 1 + 2x$$

(a) $x = 3, \; y = 13$ $\quad$ (b) $x = \frac{1}{9}, \; y = \frac{13}{9}$ $\quad$ (c) $x = 1, \; y = 3$

(d) $x = 2, \; y = 8$ $\quad$ (e) $x = 2, \; y = 10$

$y = 5x - 2$ meets $y = 1 + 2x$

when $5x - 2 = 1 + 2x$

or $3x = 3$ or $x = 1$

$\rightarrow y = 5x - 2 = 5 - 2 = 3$

$\boxed{x = 1 \quad y = 3}$

2

5 A student spending spring break in Ireland wants to visit Galway and Cork. The student has at most 7 days available and at most 500 euros to spend. Each day spent in Galway will cost 50 euros and each day spent in Cork will cost 60 euros. Let x be the number of days the student will spend in Galway and y, the number of days the student will spend in Cork. Which of the following sets of constraints describe the constraints on the student's time and money for the visits?

(a) $x + y \leq 7$, $50x + 60y \leq 500$, $x \geq 0,\ y \geq 0$

(b) $x + 7y \leq 500$, $50x + 60y \leq 1000$, $x \geq 0,\ y \geq 0$

(c) $x + y \leq 7$, $60x + 50y \leq 500$, $x \geq 0,\ y \geq 0$

(d) $x + y \geq 7$, $50x + 60y \geq 500$, $x \geq 0,\ y \geq 0$

(e) $x + y \geq 7$, $60x + 50y \geq 500$, $x \geq 0,\ y \geq 0$

	Galway (x)	Cork (y)	Available
Time.			7
Money.	50	60	500

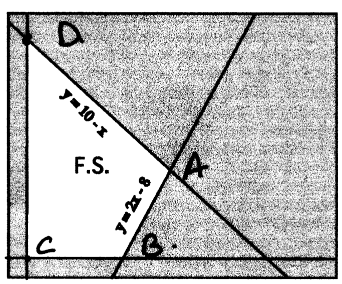
$x + y \leq 7$

$50x + 60y \leq 500$

$x \geq 0,\ y \geq 0$

6 Find the maximum value of the objective function $20x + 5y$ on the feasible set shown below.

Vertex	$20x + 5y$.
(6,4)	120 + 20 = 140
(4,0)	80 + 0 = 80
(0,0)	0 + 0 = 0
(0,10)	0 + 50 = 50



Vertices:

A. $y = 10 - x$ meets $y = 2x - 8$

or $10 - x = 2x - 8$

$18 = 3x$

$\boxed{6 = x \Rightarrow y = 4}$

B. $y = 2x - 8 = 0$

$\Rightarrow 2x = 8$ or $\boxed{x = 4 \Rightarrow y = 0}$

C: (0,0)

(a) 50 (b) 180 (c) 80 (d) 140 (e) 90

D $y = 10 - x$

$x = 0 \Rightarrow y = 10$

$\boxed{(0,10)}$

7 If A and B are the matrices given below, find $2A - 3B$.

$$A = \begin{pmatrix} 1 & 2 & 1 & 4 \\ -1 & 1 & 2 & 1 \\ 1 & -2 & 3 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 4 \\ 3 & 0 & -4 & 2 \end{pmatrix}$$

(a) $\begin{pmatrix} 2 & 2 & 1 & 3 \\ -3 & 0 & 2 & -3 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 4 & 2 & 11 \\ 4 & 5 & 4 & 14 \\ 11 & -4 & -6 & 18 \end{pmatrix}$ ~~(c)~~ $\begin{pmatrix} 5 & 4 & 2 & 5 \\ -8 & -1 & 4 & -10 \\ -7 & -4 & 18 & 6 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 2 & 1 \\ 1 & 2 & 2 \\ 4 & -2 & -1 \end{pmatrix}$ (e) $\begin{pmatrix} 5 & 4 & 2 & 5 \\ 4 & 5 & 4 & 14 \\ -2 & -2 & 7 & 4 \end{pmatrix}$

$$2A - 3B = \begin{pmatrix} 2 & 4 & 2 & 8 \\ -2 & 2 & 4 & 2 \\ 2 & -4 & 6 & 12 \end{pmatrix} - \begin{pmatrix} -3 & 0 & 0 & 3 \\ 6 & 3 & 0 & 12 \\ 9 & 0 & -12 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 4 & 2 & 5 \\ -8 & -1 & 4 & -10 \\ -7 & -4 & 18 & 6 \end{pmatrix}$$

8 Let $A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$. Which of the following is the matrix A^{-1}?

(a) $\begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 2 \\ \frac{1}{2} & -\frac{3}{2} \end{pmatrix}$ (c) $\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$ ~~(e)~~ $\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}$

$\det = 6 - 4 = 2.$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad A^{-1} = \frac{1}{\det}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$

$$= \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}$$

4

9 Rapunzel (R) and Cinderella (C) play a game where they both choose a number between 1 and 4 inclusive. Cinderella then calculates the value of Rapunzel's number minus Cinderella's number and pays Rapunzel that amount in dollars. (If this is a negative number then Cinderella will receive money from Rapunzel.) Which of the following matrices gives the payoff matrix for Rapunzel for this game?

(*a*)

		C			
	Num.	1	2	3	4
	1	4	3	2	1
	2	3	4	3	2
R	3	2	3	4	3
	4	1	2	3	4

(*b*)

		C			
	Num.	1	2	3	4
	1	0	1	2	3
	2	1	0	1	2
R	3	2	1	0	1
	4	3	2	1	0

(*c*)

		C			
	Num.	1	2	3	4
	1	1	2	3	4
	2	2	1	2	3
R	3	3	2	1	2
	4	4	3	2	1

(*d*)

		C			
	Num.	1	2	3	4
	1	0	−1	2	−3
	2	−1	0	−1	2
R	3	2	−1	0	−1
	4	−3	2	−1	0

(*e*)

		C			
	Num.	1	2	3	4
	1	0	−1	−2	−3
	2	1	0	−1	−2
R	3	2	1	0	−1
	4	3	2	1	0

R's pay-off = R# − C#

R#	C#	R's Payoff
1	1	0
2	1	1
3	1	2
4	1	3
1	2	−1 → Must be (e)

10 Romeo (R) and Collette (C) play a zero-sum game for which the payoff matrix for Romeo is given by:

	*C*1	*C*2	*C*3	*C*4	*C*5
*R*1	−1	5	9	1	4
*R*2	3	−1	−3	2	7
→ *R*3	−2	−3	1	9	8
*R*4	1	2	2	0	14

If Romeo always plays R3, which column should Collette play in order to maximize her gain?

(*a*) *C*1 (*b*) *C*2 (*c*) *C*3 (*d*) *C*4 (*e*) *C*5

Collette Should minimize R's payoff with C1

5

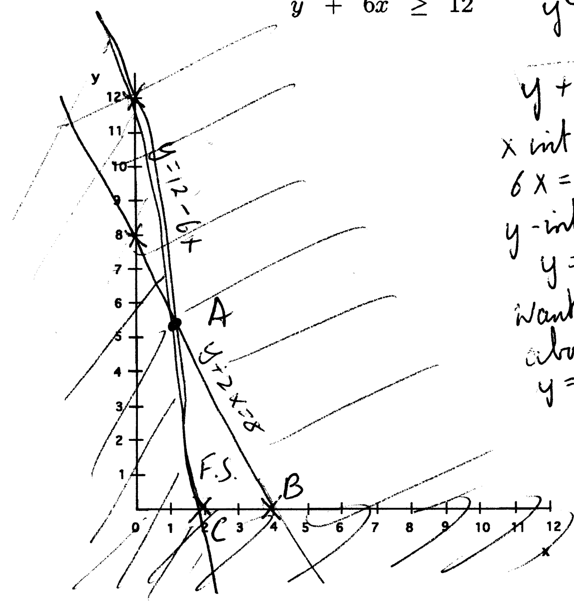
Part II, PARTIAL CREDIT, (10 Points each)

Show all of your work for credit

11, (a) Graph the feasible set corresponding to the following set of inequalities on the set of axes provided:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ y + 2x &\leq 8 \\ y + 6x &\geq 12 \end{aligned}$$

Line	st form
$\to y+2x=8$	$y \leq 8-2x$
$y+6x=12$	$y \geq 12-6x$
$y+6x=12$ x int $\Rightarrow y=0$ $6x=12 \Rightarrow x=2$ y-int $\Rightarrow x=0$ $y=12$ Want points above line $y=12-6x$	$y+2x=8$ x int $\Rightarrow y=0$ $2x=8 \Rightarrow x=4$ y-int $\Rightarrow x=0$ $y=8$ Want points below line $y+2x=8$



(b) Find the vertices of the above feasible set.

A : $y = 8-2x$ meets $y = 12-6x$. $(1, 6)$

$8-2x = 12-6x \Rightarrow 4x = 4 \Rightarrow x = 1$

$\Rightarrow y = 8-2x = 6$

B $(4, 0)$

C $(2, 0)$

(c) Find the <u>maximum</u> of the objective function $5x + 10y$ on the above feasible set.

Vertices	$5x+10y$
$(1, 6)$	$5+60=65$ *
$(4, 0)$	20
$(2, 0)$	10

Max $= 65$

6

14, Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 & 1 & 3 & 1 \\ 0 & 2 & 5 & 1 \end{pmatrix}$$

$(1,2)\begin{pmatrix} 3 \\ 5 \end{pmatrix}$

(a) Calculate the product $A \cdot B$.

$$A \cdot B = \underbrace{\begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 2 \end{pmatrix}}_{3\times 2} \underbrace{\begin{pmatrix} 4 & 1 & 3 & 1 \\ 0 & 2 & 5 & 1 \end{pmatrix}}_{2\times 4} = \underbrace{\begin{pmatrix} 4 & 5 & 13 & 3 \\ 12 & 5 & 14 & 4 \\ 8 & 6 & 16 & 4 \end{pmatrix}}_{3\times 4}$$

$(3,1)\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(b) Let

$$C = \begin{pmatrix} 1 & 5 \\ 1 & 7 \end{pmatrix}.$$

$(2,2)\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Which of the following products can be calculated?

$$A \cdot C, \quad C \cdot A, \quad B \cdot C, \quad C \cdot B.$$

$A_{3\times 2} C_{2\times 2}$	$C_{2\times 2} A_{3\times 2}$	$B_{2\times 4} C_{2\times 2}$	$C_{2\times 2} B_{2\times 4}$
Yes.	NO	NO	Yes.

(c) What is the product of the following three matrices?

$$\underbrace{(1 \;\; 0)\begin{pmatrix} 2 & 3 \\ 1 & 7 \end{pmatrix}}\begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$$= (2 \;\; 3)\begin{pmatrix} 7 \\ 1 \end{pmatrix} = 17.$$

15 (a) Cat (C) and Rat (R) play a 2-player zero-sum game, where the payoff matrix for Rat is given by the following matrix:

	C1	C2	C3	MIN
R1	2	1	0	0
R2	−2	0	−1	−2
R3	0	−3	−2	−3
Max	2	1	0	

What is Rat's optimal fixed(pure) strategy for this game ?

R1

What is Cat's optimal fixed strategy for this game?

C3

Does this pay-off matrix have a saddle point? If so where?

Yes at (R1, C3)

(b) Catherine (C) and Rasputin (R) play a 2-player zero-sum game, where the payoff matrix for Rasputin is given by the following matrix:

	C1	C2	C3	MIN
R1	−2	1	0	−2
R2	−1	0	5	−1
Max	−1	0	5	

What is Rasputin's optimal fixed(pure) strategy for this game ?

R2

What is Catherine's optimal fixed strategy for this game?

C1

Does this pay-off matrix have a saddle point? If so where?

Yes at R2C1

(c) Catman (C) and Robin (R) play a 2-player zero-sum game, where the payoff matrix for Robin is given by the following matrix:

	C1	C2	C3	MIN
R1	2	1	4	1
R2	5	−1	3	−1
R3	1	2	−5	−5
Max	5	2	4	

What is Robin's optimal fixed(pure) strategy for this game ?

R1

What is Catman's optimal fixed strategy for this game?

C2

Does this pay-off matrix have a saddle point? If so where?

NO Neither 1 nor 2 are the min in their row and the max in their column